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The gauge invariant quark correlator in QCD sum rules and lattice QCD

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Abstract: Taking the gauge invariant quark correlator as an example, in this work we perform a direct comparison of lattice QCD simulations with the QCD sum rule approach. The quark correlator is first investigated in the framework of QCD sum rules and the correlation length of the quark field is calculated. Comparing the phenomenological part of the sum rule with previous measurements of the quark correlator on the lattice, we are able to obtain an independent result for the correlation length. From a fit of the lattice data to the operator product expansion, the quark condensate and the mixed quark–gluon condensate can be extracted.

Keywords: QCD, Lattice, Sum Rules, Non–perturbative Effects.

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Contents

1. Introduction

Gauge invariant field correlators can serve as interesting examples for studying non-perturbative aspects of QCD in the framework of lattice QCD [1, 2] or QCD sum rules [3–5]. In addition, they are a natural extension of the local condensates which appear in the sum rule approach, due to the use of the operator product expansion (OPE) [6]. Phenomenological implications of nonlocal condensates have previously been discussed in the literature [7–13]. The two most fundamental correlators are the gauge invariant field strength correlator and the gauge invariant quark correlator. The gluon field strength correlator has already been investigated in lattice QCD [14–16] and QCD sum rules [17, 18]. These studies allowed to extract the correlation length of the gluon field.

On the other hand, the gauge invariant quark correlator has so far only been measured on the lattice [19, 20]. In this work, we therefore first present a QCD sum rule analysis of the gauge invariant quark correlator. To this end, the quark correlator is related to a heavy–light meson correlator [21, 22] in the heavy quark effective theory (HQET) [23]. Thus the sum rule analysis will be related to previous investigations of the same HQET correlators [24, 25] in the limit of the heavy quark

mass going to infinity. The correlation length of the quark field is then given as the inverse of the binding energy of the light quark.

In the second, and more innovative, part of the present work, we perform a direct comparison of the lattice data [19,20] with the representations of the correlation function in the QCD sum rule approach. From the so–called phenomenological parametrisation — a single resonance plus the perturbative continuum as the simplest Ansatz — we again extract the correlation length of the quark field, in agreement with the direct sum rule determination. Fitting the lattice data with the theoretical correlator in the framework of the operator product expansion, we obtain estimates of the quark and the mixed quark–gluon condensate which are in reasonable agreement with sum rule phenomenology.

Our paper is organised as follows: In the next section we discuss the relation of the gauge invariant quark correlator and the corresponding heavy quark current correlator. In section 3 we set up the expressions needed for the sum rule analysis and in section 4 we shall present our numerical results. Section 5 compares our results with recent lattice determinations of the quark correlator. In particular, a direct comparison of the OPE with the lattice data will be performed. Finally, in section 6, we conclude with a summary and an outlook.

2. The gauge invariant quark correlator

The central object of our investigation is the gauge invariant two–point correlation function of quark fields,

$$\mathcal{D}_{\alpha\beta}(z) \equiv \langle 0|T\{\bar{q}^a_\alpha(y)\,\phi^{ab}(y,x)\,q^b_\beta(x)\}|0\rangle\,, \tag{2.1}$$

where the string operator $\phi^{ab}(y,x)$ ensures the gauge invariance of the correlator and is represented by

$$\phi^{ab}(y,x) \equiv [\,\mathcal{P}\,e^{igT^C\int_0^1 d\lambda\, z^\mu A^C_\mu(x+\lambda z)}\,]^{ab}\,, \tag{2.2}$$

with $z = y - x$. T^C are the generators of SU(3) in the fundamental representation and $\mathcal{P}$ denotes path ordering of the exponential. In general, the gauge invariant field strength correlator could be defined using an arbitrary gauge string connecting the end points x and y, but in this work we have restricted ourselves to a straight line, for it is only in this case that the correlator (2.1) is related to a heavy meson correlator in HQET.

Using the path–integral formalism, we are able to derive a relation between the correlator $\mathcal{D}_{\alpha\beta}(z)$ and the correlator of a local, gauge invariant current composed of a light quark field $q^a_\alpha(x)$ and an infinitely heavy quark field $h^a_\alpha(x)$ [22]. Analogously

to HQET, the heavy quark field $h^a_\alpha(x)$ is constructed from the field $Q^a_\alpha(x)$ with a finite mass m_Q in the limit $m_Q \to \infty$,

$$h^a_\alpha(x) \;=\; \lim_{m_Q\to\infty} \frac{1}{2}\,(1+\not{v})_{\alpha\beta}\, e^{im_Q vx}\, Q^a_\beta(x)\,, \tag{2.3}$$

where v_μ is the four–velocity of the heavy quark. In the case of infinitely heavy quarks, the coupling to the gauge fields is given by the effective HQET action $S_{eff} = \int d^4x\, \bar{h}\, iv^\mu D_\mu h$ with $D_\mu = \partial_\mu - igA_\mu$ [23]. In the free field case the heavy quark propagator is given by

$$\begin{aligned} S^{ab}_{\alpha\beta}(z) &\equiv \langle 0|T\{h^a_\alpha(y)\,\bar{h}^b_\beta(x)\}|0\rangle = \delta^{ab}\,\frac{1}{2}\,(1+\not{v})_{\alpha\beta}\,S(z) \\ &= \delta^{ab}\,\frac{1}{2}\,(1+\not{v})_{\alpha\beta}\,\frac{1}{v^0}\,\theta(z^0)\,\delta\Big(\mathbf{z} - \frac{z^0}{v^0}\mathbf{v}\Big)\,, \end{aligned} \tag{2.4}$$

where v^0 is the zero–component of the velocity. In addition, we have the relation $v_\mu = z_\mu/|z|$ with $|z| \equiv \sqrt{z^2 - i\epsilon}$ which follows immediately from the constraints $\mathbf{z} = z^0\mathbf{v}/v^0$ and $v^2 = 1$. In order to obtain a solution for the interacting case, we have to find the inverse of the operator $iv^\mu D_\mu$. Using the following crucial relation obeyed by the string operator,

$$v^\mu \partial^y_\mu\, \phi(y,x) = v^\mu\, igT^C A^C_\mu(y)\,\phi(y,x)\,, \tag{2.5}$$

together with equation (2.4) one can show that the solution for the interacting case is found to be [22]

$$\langle 0|T\{h^a_\alpha(y)\,\bar{h}^b_\beta(x)\,e^{iS_{eff}}\}|0\rangle = S_{\alpha\beta}(z)\,\langle 0|[\,\mathcal{P}e^{igT^C\int_0^1 d\lambda\, z^\mu A^C_\mu(x+\lambda z)}\,]^{ab}|0\rangle\,. \tag{2.6}$$

The physical interpretation of this result is an infinitely heavy quark moving from point x to point y with a four–velocity v, acquiring a phase proportional to the path–ordered exponential. The limit $m_Q \to \infty$ is necessary in order to constrain the heavy quark to a straight line and to decouple the spin interactions.

The result (2.6) can be employed to relate the correlator (2.1) to correlators of gauge invariant local currents in HQET. To this end, we define the pseudoscalar and scalar heavy meson currents as

$$j_P(x) = \bar{q}^a_\alpha(x)\,(i\gamma_5)_{\alpha\beta}\,h^a_\beta(x) \quad \text{and} \quad j_S(x) = \bar{q}^a_\alpha(x)\,h^a_\alpha(x)\,, \tag{2.7}$$

as well as the correlators $\widetilde{\mathcal{D}}_\Gamma(z)$ with $\Gamma = P$ or S:

$$\widetilde{\mathcal{D}}_\Gamma(z) \equiv \langle 0|T\{j_\Gamma(y)\,j^\dagger_\Gamma(x)\}|0\rangle\,. \tag{2.8}$$

The two correlators could have also been defined with vector and axialvector currents, but as it was shown in ref. [24], the vector and axialvector correlators are equal to

the pseudoscalar and scalar correlators respectively in the heavy mass limit. This result is related to the fact that in the heavy mass limit the corresponding physical states become degenerate. Integrating out the heavy quark fields in (2.8), we obtain the expressions

$$\begin{aligned}\widetilde{\mathcal{D}}_P(z) &= -\frac{1}{2}\,\langle 0|\bar{q}^a(y)(1-\not{v})\phi^{ab}(y,x)q^b(x)|0\rangle\, S(z)\,,\\ \widetilde{\mathcal{D}}_S(z) &= \;\;\,\frac{1}{2}\,\langle 0|\bar{q}^a(y)(1+\not{v})\phi^{ab}(y,x)q^b(x)|0\rangle\, S(z)\,,\end{aligned}\tag{2.9}$$

displaying the relation of the heavy meson correlators to the gauge invariant quark correlator of equation (2.1).

In energy space the HQET correlator $\widetilde{\mathcal{D}}_\Gamma(w)$ obeys the spectral representation

$$\widetilde{\mathcal{D}}_\Gamma(w) = \sum_k \frac{f_{\Gamma,k}^2}{(E_{\Gamma,k}-w-i\epsilon)} + \int_{w_0^\Gamma}^{\infty} d\lambda\,\frac{\rho_\Gamma(\lambda)}{(\lambda-w-i\epsilon)}\,,\tag{2.10}$$

where $w = vq$ and the Fourier transform of the correlators $\widetilde{\mathcal{D}}_\Gamma(z)$ in coordinate space is given by

$$\widetilde{\mathcal{D}}_\Gamma(w) = i\int d^4z\, e^{iqz}\,\widetilde{\mathcal{D}}_\Gamma(z)\,.\tag{2.11}$$

$E_{\Gamma,k}$ represents the energy of the bound states and $f_{\Gamma,k}$ is the coupling of the state k with quantum numbers Γ to the vacuum,

$$\langle 0|j_\Gamma(0)|H_{\Gamma,k}\rangle = f_{\Gamma,k}\,.\tag{2.12}$$

The spectral densities are defined by $\rho_\Gamma(\lambda) \equiv 1/\pi\,\mathrm{Im}\,\widetilde{\mathcal{D}}_\Gamma(\lambda+i\epsilon)$, and finally w_0^Γ is the threshold energy of the continuum contribution. Transforming the spectral representation (2.10) back to coordinate space, one obtains

$$\begin{aligned}\widetilde{\mathcal{D}}_\Gamma(z) &= -i\int \frac{d^4q}{(2\pi)^4}\, e^{-iqz}\,\widetilde{\mathcal{D}}_\Gamma(w) \equiv S(z)\,\mathcal{D}_\Gamma(z)\\ &= S(z)\left\{\sum_k f_{\Gamma,k}^2 e^{-iE_{\Gamma,k}|z|} + \int_{w_0^\Gamma}^{\infty} d\lambda\,\rho_\Gamma(\lambda)\, e^{-i\lambda|z|}\right\}.\end{aligned}\tag{2.13}$$

The factorisation of the heavy quark propagator and the relations (2.9) ensure a representation of the gauge invariant quark correlators $\mathcal{D}_\Gamma(z)$ as given by the expression inside the curly brackets. Let us already remark that the correlator decays as a simple exponential in the Euclidean region. The correlation length will therefore be given by the inverse of the lowest lying bound state energy.

3. The sum rules

We now turn to the theoretical side of the sum rules which arises from calculating the correlator of equation (2.8) in the framework of the operator product expansion [3,6]. The perturbative contributions for the spectral density up to next–to–leading order in the strong coupling constant have been calculated in [24]. Up to $\mathcal{O}(m^2)$ in the light quark mass, they are found to be:

$$
\begin{aligned}
\rho_\Gamma^{pt}(w) &= w^2 \sum_{n=0}^{2} \Big[\, d_{n0}^\Gamma + a(d_{n1}^\Gamma + d_{n1L}^\Gamma L) \Big] \Big(\frac{m}{w}\Big)^n \\
&= \frac{N_c}{8\pi^2} \bigg\{ w^2 \Big[4 + C_F a \Big(17 + \tfrac{4}{3}\pi^2 - 6L\Big) \Big] \\
&\qquad \pm wm \Big[4 + C_F a \Big(24 + \tfrac{4}{3}\pi^2 - 12L\Big) \Big] - m^2 \Big[2 + C_F a \Big(3 - 9L\Big) \Big] \bigg\}
\end{aligned}
\tag{3.1}
$$

where $a \equiv \alpha_s/\pi$ and $L \equiv \ln(2w/\mu)$. Equation (3.1) also defines the coefficients d_{ni}^Γ which will be utilised below. As above and in all the following, the upper sign corresponds to the pseudoscalar and the lower sign to the scalar current.

In order to suppress contributions in the dispersion integral coming from higher exited states and from higher dimensional operators, it is convenient to apply a Borel transformation $\widehat{B}_T$ with T being the Borel variable. The Borel transformation also removes the subtraction constants which are present in the dispersion relation satisfied by the correlators. Some useful formulae for the Borel transformation are collected in the appendix. For the phenomenological side of the sum rules, equation (2.10), we only take the lowest lying resonance and approximate the spectral density by the perturbative expression (3.1), assuming quark–hadron duality for $w > w_0^\Gamma$. We then obtain

$$
\widehat{\mathcal{D}}_\Gamma(T) \;=\; f_\Gamma^2\, e^{-E_\Gamma/T} + \int\limits_{w_0^\Gamma}^{\infty} d\lambda\, \rho_\Gamma^{pt}(\lambda)\, e^{-\lambda/T}\,. \tag{3.2}
$$

Let us remark that equation (3.2) takes exactly the same form as the expression of (2.13) inside the curly brackets, which represents the gauge invariant quark correlator, if we identify $1/T$ with the Euclidean space-time coordinate.

The perturbative contribution relevant for the sum rules is the full correlator minus the corresponding continuum contribution:

$$
\widehat{\mathcal{D}}_\Gamma^{pt}(T) - \widehat{\mathcal{D}}_\Gamma^{co}(T, w_0^\Gamma) \;=\; \int\limits_0^{\infty} d\lambda\, \rho_\Gamma^{pt}(\lambda)\, e^{-\lambda/T} - \int\limits_{w_0^\Gamma}^{\infty} d\lambda\, \rho_\Gamma^{pt}(\lambda)\, e^{-\lambda/T}\,, \tag{3.3}
$$

where $\widehat{\mathcal{D}}_\Gamma^{co}(T, w_0^\Gamma)$ denotes the perturbative continuum part. After the Borel transformation the correlators satisfy homogeneous renormalisation group equations. Thus

we can improve the perturbative expressions by resumming the logarithmic contributions. With the help of the following general integral formula [26],

$$\int\limits_{w_0^\Gamma}^{\infty} d\lambda\,\lambda^{\alpha-1}\ln^n\frac{2\lambda}{\mu}\,e^{-\lambda/T} \;=\; T^\alpha\sum_{k=0}^{n}\binom{n}{k}\ln^k\frac{2T}{\mu}\left[\frac{\partial^{n-k}}{\partial\alpha^{n-k}}\,\Gamma\!\left(\alpha,\,\frac{w_0^\Gamma}{T}\right)\right], \tag{3.4}$$

the perturbative contribution to the sum rules is found to be:

$$\begin{aligned}&\widehat{\mathcal{D}}_\Gamma^{pt}(T)-\widehat{\mathcal{D}}_\Gamma^{co}(T,w_0^\Gamma) \;= \\ &T^3\left(\frac{a(2T)}{a(\mu)}\right)^{\gamma_1/\beta_1}\sum_{n=0}^{2}\phi(3-n,y)\left[d_{n0}^\Gamma+a\left(d_{n1}^\Gamma+d_{n1L}^\Gamma\frac{\phi'(3-n,y)}{\phi(3-n,y)}\right)\right]\left(\frac{m(2T)}{T}\right)^n\end{aligned} \tag{3.5}$$

where $\phi(\alpha,y)\equiv\Gamma(\alpha)-\Gamma(\alpha,y)$, $\phi'(\alpha,y)\equiv\partial/\partial\alpha\,\phi(\alpha,y)$ and $y=w_0^\Gamma/T$. Some explicit expressions for the incomplete Γ-function $\Gamma(\alpha,y)$ and the functions $\phi(\alpha,y)$, $\phi'(\alpha,y)$ can also be found in the appendix. In our notation $\beta_1=(11N_c-2N_f)/6$ is the first coefficient of the QCD β-function. The anomalous dimension γ_1 of both currents is easily found by reexpanding the running coupling and mass in terms of $a(\mu)$ and $m(\mu)$. The resulting expression is

$$\gamma_1 \;=\; -\,\frac{d_{n1L}^\Gamma}{d_{n0}^\Gamma}-n\,\gamma_{1m} \;=\; \frac{3}{2}\,C_F \;=\; 2\,, \tag{3.6}$$

where the first coefficient of the mass anomalous dimension, $\gamma_{1m}=3C_F/2$, has been used. The μ-dependence of the correlators is due to the fact that the pseudoscalar and scalar currents are not renormalisation group invariant quantities. However, the product of m times the current, $m\cdot j_\Gamma$, is renormalisation group invariant in the full theory. Taking into account the additional multiplicative renormalisation of the heavy quark current in HQET [24, 27], one again finds $\gamma_1=\gamma_{1m}$.

Essential for QCD sum rule analyses are the non–perturbative contributions coming from vacuum condensates. In the operator product expansion, the correlation function is expanded in powers of $1/w$ corresponding to higher and higher dimensional condensates. In our case the dimension three condensate $\langle\bar hh\rangle$ vanishes since the heavy quark mass is infinite. After Borel transformation and up to operators of dimension five, the non–perturbative contribution takes the form [24]:

$$\widehat{\mathcal{D}}_\Gamma^{np}(T) \;=\; \mp\,\frac{\langle\bar qq\rangle_\mu}{2}\left[1\mp\frac{m}{4T}+\frac{3}{2}\,C_F a\right]\pm\frac{\langle g\bar q\sigma Gq\rangle_\mu}{32T^2}\,. \tag{3.7}$$

The next condensate contribution would be of dimension six. We have checked explicitly in our numerical analysis that this contribution to the sum rule is small. Thus we shall neglect all condensate contributions for dimensions higher than five

in this work. For consistency, we have also omitted the known order $\mathcal{O}(a^2)$ contribution to the quark condensate [24], because the corresponding corrections to the perturbative part have not yet been calculated. We have, however, verified that also this correction only has a minor impact on our numerical results.

In the case of the non–perturbative part, the scale dependence of the correlator is implicit in the μ-dependence of the quark condensate. Indeed, again $m\langle\bar{q}q\rangle$ is scale independent and therefore $\langle\bar{q}q\rangle$ scales inversely like the quark mass yielding the same μ-dependence as for the perturbative contribution. For the mixed quark–gluon condensate, the next–to–leading order corrections have not been calculated and thus in the numerical analysis below, we shall neglect its scale dependence.

4. Numerical analysis

After equating the phenomenological and the theoretical contributions to the correlation functions, we end up with the following sum rule:

$$K_\Gamma(T) \;\equiv\; f_\Gamma^2\, e^{-E_\Gamma/T} \;=\; \widehat{\mathcal{D}}_\Gamma^{pt}(T) - \widehat{\mathcal{D}}_\Gamma^{co}(T,w_0^\Gamma) + \widehat{\mathcal{D}}_\Gamma^{np}(T)\,. \tag{4.1}$$

The binding energy E_Γ can be obtained by dividing the derivative of the sum rule (4.1) with respect to $-1/T$ by the original sum rule:

$$\begin{aligned} E_\Gamma \;&=\; -\,\frac{\partial}{\partial(1/T)}\,\ln K_\Gamma \\ &=\; -\,\frac{\frac{\partial}{\partial(1/T)}\Big(\widehat{\mathcal{D}}_\Gamma^{pt}(T) - \widehat{\mathcal{D}}_\Gamma^{co}(T,w_0^\Gamma) + \widehat{\mathcal{D}}_\Gamma^{np}(T)\Big)}{\Big(\widehat{\mathcal{D}}_\Gamma^{pt}(T) - \widehat{\mathcal{D}}_\Gamma^{co}(T,w_0^\Gamma) + \widehat{\mathcal{D}}_\Gamma^{np}(T)\Big)}\,. \end{aligned} \tag{4.2}$$

Analogously to equation (3.5) an expression for the derivative of the perturbative contribution can be obtained with the help of formula (3.4):

$$\begin{aligned} &-\frac{\partial}{\partial(1/T)}\Big(\widehat{\mathcal{D}}_\Gamma^{pt}(T) - \widehat{\mathcal{D}}_\Gamma^{co}(T,w_0^\Gamma)\Big) \;= \\ &T^4\left(\frac{a(2T)}{a(\mu)}\right)^{\gamma_1/\beta_1}\sum_{n=0}^{2}\phi(4-n,y)\left[d_{n0}^\Gamma + a\left(d_{n1}^\Gamma + d_{n1L}^\Gamma\,\frac{\phi'(4-n,y)}{\phi(4-n,y)}\right)\right]\left(\frac{m(2T)}{T}\right)^n\,. \end{aligned} \tag{4.3}$$

The corresponding derivative of the non–perturbative contribution is easily calculated from equation (3.7). Because the renormalisation of the correlators is multiplicative, it is clear that the binding energy E_Γ is a physical quantity in the sense that it is scale and scheme independent. On the contrary, this is not the case for the decay constant f_Γ, as we shall discuss in more detail below.

Let us first consider the pseudoscalar correlator for a vanishing light quark mass $m_q = 0$. As the central values for our input parameters we use $\langle\bar{q}q\rangle(1\,\mathrm{GeV}) = -(235\pm$

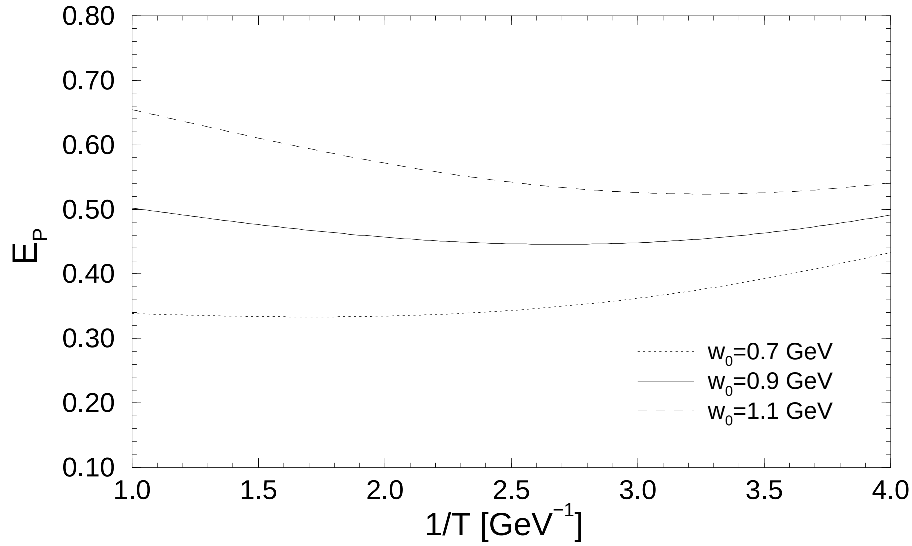

Figure 1: Pseudoscalar binding energy as a function of $1/T$ for different values of the continuum threshold w_0^P.

$20\,\mathrm{MeV})^3$ for the quark condensate, $\langle g\bar{q}\sigma G q\rangle = M_0^2\,\langle\bar{q}q\rangle$ with $M_0^2 = (0.8\pm 0.2)\,\mathrm{GeV}^2$ [28–30] for the mixed condensate and $\Lambda_{\overline{MS}}^{(3)} = 340\,\mathrm{MeV}$. The latter value corresponds to three light quark flavours and $\alpha_s(M_Z) = 0.119$. In principle, the coupling constant in the next–to–leading order term could be evaluated at any scale μ. As our central value in the numerical analysis we have chosen $\mu = 1.2\,\mathrm{GeV}$ but we shall investigate the variation of μ below. In figure 1, we display the pseudoscalar binding energy E_P as a function of $1/T$ for different values of the continuum threshold w_0^P. A sum rule window, were both the continuum as well as the condensate contributions are reasonably small, can be found around $1/T = 2.5 - 3.0\,\mathrm{GeV}^{-1}$. As can be seen from figure 1, best stability is achieved for $w_0^P \approx 0.9\,\mathrm{GeV}$. Estimating the value of E_P in the given range $0.7\,\mathrm{GeV} < w_0^P < 1.1\,\mathrm{GeV}$, we obtain:

$$E_P \;=\; 450 \pm 100\,\mathrm{MeV} \qquad (m_q \;=\; 0)\,. \tag{4.4}$$

Although the next–to–leading order QCD corrections to the sum rule of equation (4.1) are very large, of the order of 100%, in the ratio of equation (4.3) they cancel to a large extent. To investigate the sensitivity of our result to higher order corrections, we next study the dependence on the renormalisation scale μ. The dependence of E_P on the renormalisation scale μ is shown in figure 2 for $w_0^P = 0.9\,\mathrm{GeV}$ and $\mu = 0.8\,\mathrm{GeV}$, $1.2\,\mathrm{GeV}$ and $2.4\,\mathrm{GeV}$. One should not take μ smaller than $0.8\,\mathrm{GeV}$ because then also the radiative corrections to the quark condensate become unacceptably large. We observe that inspite of the huge corrections to the correlation function, the scale dependence of E_P is relatively mild. Adding this uncertainty to

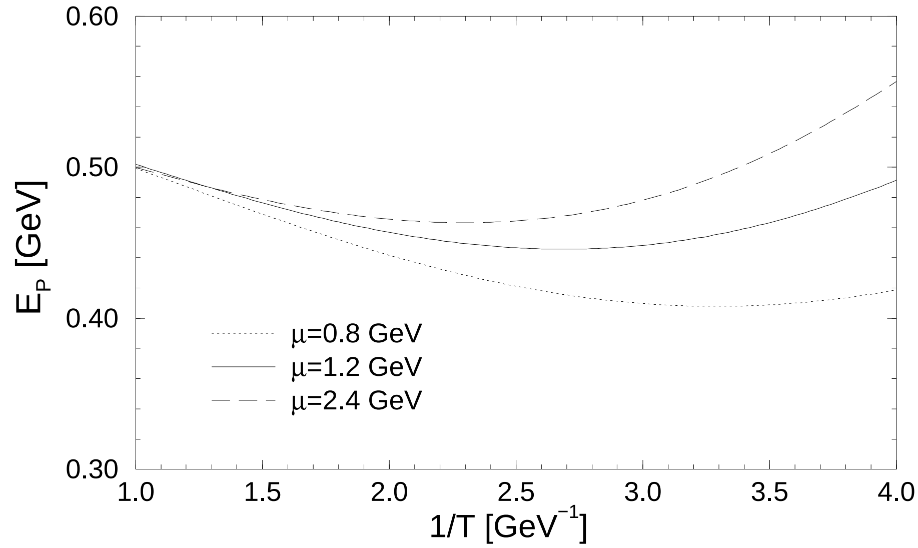

Figure 2: Pseudoscalar binding energy as a function of $1/T$ for $w_0^P = 0.9\,\mathrm{GeV}$ and different values of the renormalisation scale μ.

the error on E_P, our central result for E_P is found to be

$$E_P \;=\; 450 \pm 150\,\mathrm{MeV} \qquad (m_q \;=\; 0)\,. \tag{4.5}$$

The additional uncertainty coming from the errors on the input parameters is small compared to the estimated uncertainty and can be neglected.

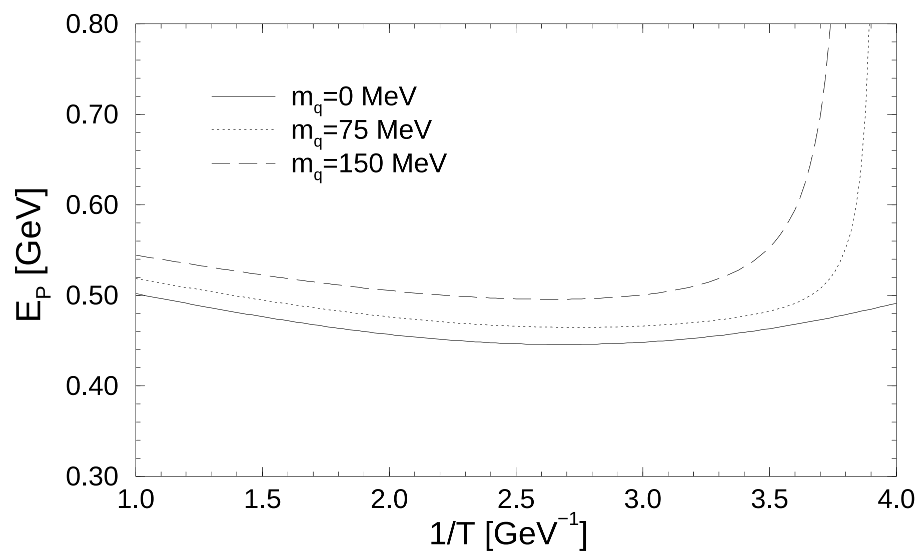

Figure 3: Pseudoscalar binding energy as a function of $1/T$ for $w_0^P = 0.9\,\mathrm{GeV}$, $\mu = 1.2\,\mathrm{GeV}$ and different values of the light quark mass m_q. The absolute value of $\langle \bar{q}q \rangle$ has been reduced in accord with the quark mass.

Let us now investigate the influence of a finite light quark mass m_q. In figure 3, we have plotted E_P as a function of $1/T$ for $w_0^P = 0.9\,\text{GeV}$, $\mu = 1.2\,\text{GeV}$ and three values of the light quark mass m_q, namely $m_q = 0\,\text{MeV}$, $75\,\text{MeV}$ and $150\,\text{MeV}$. The last value is in the region of the mass of the strange quark and the intermediate value is interesting for comparison with lattice QCD results. It is well known from QCD sum rules that at the mass of the strange quark, the absolute value of the corresponding quark condensate is reduced by roughly 30% [4]. This reduction has been applied for obtaining the dashed curve in figure 3. Lacking further information, for the dotted curve corresponding to $m_q = 75\,\text{MeV}$ the quark condensate has been reduced by 10%.

Qualitatively, we find that the binding energy increases with increasing light quark mass and decreasing quark condensate. For $m_q = 150\,\text{MeV}$ this increase turns out to be of the order of $50\,\text{MeV}$. Such a value is only about half of the mass splitting of B and B_s mesons in the B-meson system as well as D and D_s mesons in the D-meson system which is of the order of $100\,\text{MeV}$. Further comparison with the heavy meson systems will be presented below. The instability for $1/T \gtrsim 3.5\,\text{GeV}^{-1}$ results from the running quark mass $m_q(2T)$ which explodes in this region due to uncontrollably large higher order corrections.

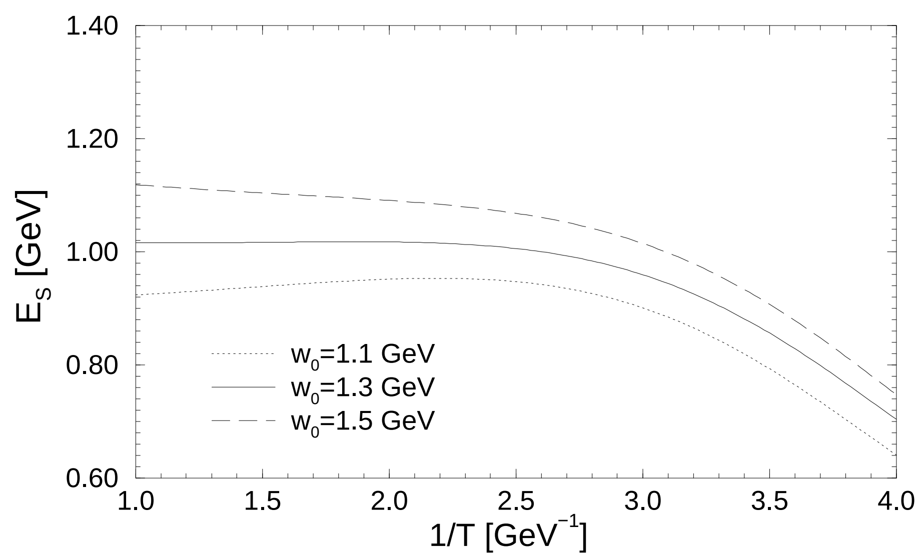

Figure 4: Scalar binding energy as a function of $1/T$ for different values of the continuum threshold w_0^S.

Let us next turn to the scalar correlator. In figure 4 the scalar binding energy is shown for three different values of the continuum threshold w_0^S. We observe that generally the stability of the scalar sum rule as not as good as in the pseudoscalar

case. Here, the region of best stability is around $1/T = 2.0\,\mathrm{GeV}^{-1}$. Adding the uncertainty from the scale dependence, the scalar binding energy is found to be:

$$E_S \,=\, 1.0 \pm 0.2\,\mathrm{GeV} \qquad (m_q \,=\, 0)\,. \tag{4.6}$$

Compared to the pseudoscalar correlator, the dependence of E_S on the light quark mass m_q is somewhat stronger. Approximately, we obtain $E_S(m_q) \approx E_S(0) + m_q$ for $m_q \leq 150\,\mathrm{MeV}$. However, in the scalar case for increasing quark mass and decreasing quark condensate, the sum rule becomes less stable and thus we refrain from making more quantitative statements.

To conclude this section, let us comment on the decay constants f_Γ. As has been already discussed above, because of the renormalisation of the heavy meson current, f_Γ depends on the renormalisation scale and scheme. Therefore, it is convenient to define renormalisation-group invariant decay constants $\widehat{f}_\Gamma$. At the next–to–leading order, $\widehat{f}_\Gamma$ takes the form [24, 25]:

$$\widehat{f}_\Gamma \,=\, f_\Gamma\, \alpha_s(\mu)^{\gamma_1/2\beta_1}(\,1 + K\,a\,) \quad \text{with} \quad K \,=\, \frac{5}{12} - \frac{285 - 7\pi^2}{54\beta_1} + \frac{107}{8\beta_1^2}\,. \tag{4.7}$$

As central values, from the sum rule of equation (4.1) we then find $\widehat{f}_P = 0.3\,\mathrm{GeV}^{3/2}$ and $\widehat{f}_S = 0.5\,\mathrm{GeV}^{3/2}$. Since the next–to–leading QCD correction to the correlators is very large, this is also true for the uncertainty on $\widehat{f}_\Gamma$. Thus we shall not dicuss the heavy meson decay constants further.

5. Comparison with lattice results

In ref. [19] the two following quark–antiquark nonlocal condensates have been determined on the lattice:

$$\begin{aligned} C_0(x) &= -\sum_{f=1}^{4} \langle \mathrm{Tr}[\bar{q}_\alpha^f(0)\phi(0,x)q_\alpha^f(x)]\rangle\,, \\ C_v(x) &= -\frac{x_\mu}{|x|}\sum_{f=1}^{4} \langle \mathrm{Tr}[\bar{q}_\alpha^f(0)(\gamma_E^\mu)_{\alpha\beta}\phi(0,x)q_\beta^f(x)]\rangle\,. \end{aligned} \tag{5.1}$$

The trace in (5.1) is taken with respect to the colour indices and f is a flavour index for the light quarks. γ_E^μ are Euclidean Dirac matrices, defined as: $\gamma_{E4} = \gamma^0$, $\gamma_{Ei} = -i\gamma^i$ $(i = 1, 2, 3)$. Other insertions of gamma matrices than those containing 1 and γ_E^μ vanish by P and T invariance.

In order to avoid confusion with the correlators introduced in section 2 the correlators $C_0(x)$ and $C_v(x)$ are called respectively "spin–zero" nonlocal condensate and

"longitudinal–vector" nonlocal condensate. They are simply related to the correlators $\widetilde{\mathcal{D}}_\Gamma$ of equation (2.9) if the latter are continued to Euclidean space time:

$$\widetilde{\mathcal{D}}_P(x) = \frac{1}{2N_f}\Big(C_v(x) + C_0(x)\Big)\,S(x)\,,$$
$$\widetilde{\mathcal{D}}_S(x) = \frac{1}{2N_f}\Big(C_v(x) - C_0(x)\Big)\,S(x)\,, \tag{5.2}$$

where N_f is the number of quark flavours.

The lattice computations of ref. [19] have been performed both in the *quenched* approximation and in full QCD using the SU(3) Wilson action for the pure–gauge sector and four degenerate flavours of *staggered* fermions, so that the sum over the flavour index f goes from 1 to 4. In full QCD the nonlocal condensates have been measured on a $16^3 \times 24$ lattice at $\beta = 5.35$ and two different values of the quark mass: $a \cdot m_q = 0.01$ and $a \cdot m_q = 0.02$ (a being the lattice spacing). For the *quenched* case the measurements have been performed on a 16^4 lattice at $\beta = 6.00$, using valence quark masses $a \cdot m_q = 0.01$, 0.05, 0.10, and at $\beta = 5.91$ with a quark mass $a \cdot m_q = 0.02$. Further details, as well as a remark about the reliability of the results obtained for the longitudinal–vector nonlocal condensate, can be found in [19]. The scalar functions C_0 and C_v introduced in ref. [19] have a more complicated spectral representation than $\widetilde{\mathcal{D}}_P$ and $\widetilde{\mathcal{D}}_S$, since they receive contributions both from scalar and pseudoscalar intermediate states. Nevertheless, the correlator C_0 has the advantage of not receiving perturbative contributions in the chiral limit $m_q \to 0$ with m_q being the mass of the light quark:

$$\lim_{x\to 0}\; C_0(x) \to \frac{N_f N_c m_q}{\pi^2 x^2} \qquad \text{and} \qquad \lim_{x\to 0}\; C_v(x) \to \frac{2N_f N_c}{\pi^2 x^3}\,, \tag{5.3}$$

where N_c is the number of colours.

Since the available lattice results for $C_v(x)$ are not conclusive, in what follows we shall concentrate on the spin–zero nonlocal condensate $C_0(x)$. In ref. [19] a best fit to the data with the following function has been performed:

$$C_0(x) \;=\; A_0 \exp(-\mu_0 x) + \frac{B_0}{x^2}\,. \tag{5.4}$$

The perturbative–like term B_0/x^2 takes the form obtained in the leading order in perturbation theory, if the chiral limit $m_q \to 0$ (see equation (5.3)) is approached. Using the ansatz (5.4), the correlation length $\lambda_0 \equiv 1/\mu_0$ of the spin–zero quark–antiquark nonlocal condensate can be extracted. At the lightest quark mass $a \cdot m_q = 0.01$ in full QCD one obtains the value $\lambda_0 = 0.63^{+0.21}_{-0.13}$ fm [19] corresponding to $\mu_0 = 310 \pm 80\,\mathrm{MeV}$. Within errors this value agrees with the value for E_P obtained from the sum rule analysis.

Some observations are, however, necessary at this point. One should point out that the parametrisation (5.4) is a sort of "hybrid" parametrisation, since it contains a "perturbative" term B_0/x^2, which should reproduce the behaviour predicted by perturbation theory at short distances, and a "non–perturbative" term $A_0 \exp(-\mu_0 x)$, which should reproduce the predicted exponential behaviour at large distances. A simple exponential term is not dominant in the range of physical distances where lattice data are taken, i.e., $x \approx 0.2 \div 0.8$ fm. This could shed some doubt on the identification of $\mu_0 = 1/\lambda_0$ with the binding energy E_P, as determined in section 4.

Nevertheless, at the distances where lattice data are taken the operator product expansion should still be a reasonable approximation. One can therefore try to fit directly the OPE expression to the lattice data. In this case however a running coupling would lead to a Landau pole around 1 fm and one should confine the comparison to distances small compared to this scale. On the other hand most treatments of non–perturbative QCD are based on the assumption that at large distances only non–perturbative effects prevail. This can be achieved by freezing the coupling at a certain value for distances larger than a critical one. Given the fact that we have only few data points we consider here only the leading order terms in the OPE. In addition, the correlator is scheme dependent and to perform a consistent comparison between the lattice data and the OPE at the next–to–leading order, a perturbative calculation in the lattice scheme would be necessary.

At the leading order the spin–zero correlator $C_0(x)$ up to operators of dimension seven is given by:

$$C_0(x)/N_f \;=\; \frac{N_c m_q^2}{\pi^2 x}\, K_1(m_q x) - \Big[\, 1 + \tfrac{1}{8}\, m_q^2 x^2 \,\Big] \langle \bar{q}q \rangle + \frac{x^2}{16}\, \langle g\bar{q}\sigma G q \rangle \,, \tag{5.5}$$

where in the perturbative part through the Bessel function $K_1(z)$ we have kept all orders in the quark mass. Fits for the quark and the mixed condensate from the different sets of lattice data of ref. [19] are given in table 1. The presented errors just correspond to the statistical $1\,\sigma$ errors resulting from the fit if the $\chi^2/d.o.f.$ is normalised to one. m_L is the lattice mass converted to physical units and m_q the mass appearing in the OPE of equation (5.5). Since we work at the leading order, the scale and scheme dependence of the quark mass are not under control.

We note that the condensates come out with the correct order of magnitude. Qualitatively also the decrease of the quark condensate with increasing quark mass is found albeit somewhat stronger than expected from phenomenology [4]. A direct extraction of the quark condensate from the uncooled values of the spin–zero quark correlator [19] at zero distance led to the value $\langle \bar{q}q \rangle (1\,\mathrm{GeV}) = -\,(235 \pm 15\,\mathrm{MeV})^3$ in perfect agreement with phenomenological determinations. We have again parametrised

β	$a \cdot m$	m_L [MeV]	m_q [MeV]	$-\langle \bar{q}q \rangle^{1/3}$ [MeV]	M_0^2 [GeV^2]
6.00, q	0.01	19	38 ± 4	244 ± 4	0.47 ± 0.03
5.91, q	0.02	33	73 ± 11	198 ± 12	0.44 ± 0.06
6.00, q	0.05	96	150 ± 14	186 ± 15	0.68 ± 0.09
6.00, q	0.10	192	276 ± 22	175 ± 20	0.87 ± 0.12
5.35, f	0.01	20	28 ± 2	168 ± 4	0.47 ± 0.04
5.35, f	0.02	33	54 ± 6	177 ± 9	0.45 ± 0.04

Table 1: Determination of the condensates by direct comparison of lattice data with the OPE. q denotes quenched approximation, f full QCD with 4 light fermions, m_L is the input lattice mass and m_q the OPE mass appearing in equation (5.5).

the mixed quark–gluon condensate through $\langle g\bar{q}\sigma Gq \rangle = M_0^2 \langle \bar{q}q \rangle$. The dependence of M_0^2 on the quark mass is controversial in the literature [31, 32]. From our fits we obtain an increase of M_0^2 with increasing quark mass, supporting the findings of ref. [31].

The non–perturbative part of the OPE (5.5) can be expressed at short distances by the Gaussian $-\langle \bar{q}q \rangle \exp(-M_0^2 x^2/16)$ which at large distances displays an exponential falloff. Fitting the lattice data for the quenched calculation with $a \cdot m_q = 0.01$ to the perturbative part of (5.5) and the Gaussian non–perturbative contribution, we find $m_q = 33 \pm 2\,\text{MeV}$, $-\langle \bar{q}q \rangle^{1/3} = 254 \pm 2\,\text{MeV}$ and $M_0^2 = 0.74 \pm 0.02\,\text{GeV}^2$. This value for M_0^2 is more compatible with phenomenological determinations [28–30]. However, the result shows that higher order corrections in the OPE have some importance and it gives an indication about the systematic uncertainties.

Another way to improve the hybrid expansion (5.4) consists in taking the higher resonances into account in the same way as it is done in the sum rule technique as developed by SVZ [3], namely by including the perturbative continuum above a threshold w_0. In leading order for the spin–zero nonlocal condensate this yields:

$$\begin{aligned} C_0(x)/N_f = {} & f_P^2\, e^{-E_P x} - f_S^2\, e^{-E_S x} \\ & + \frac{N_c}{2\pi^2 x^3} \Big((2 + 2w_0^P x + (w_0^P x)^2) e^{-w_0^P x} - (2 + 2w_0^S x + (w_0^S x)^2) e^{-w_0^S x} \Big) \\ & + \frac{N_c m_q}{2\pi^2 x^2} \Big((1 + w_0^P x) e^{-w_0^P x} + (1 + w_0^S x) e^{-w_0^S x} \Big) . \end{aligned} \tag{5.6}$$

In figure 5 we have displayed the lattice data for the quenched calculation with $a \cdot m_q = 0.01$. The dashed curve is the fit with the OPE (5.5) and the parameters given in the first row of table 1. The solid curve is a fit with equation (5.6) where for simplicity, and lacking enough data points, we have neglected the scalar resonance by setting $f_S = 0$ and $w_0^S = 0$. This means that in the scalar channel only the

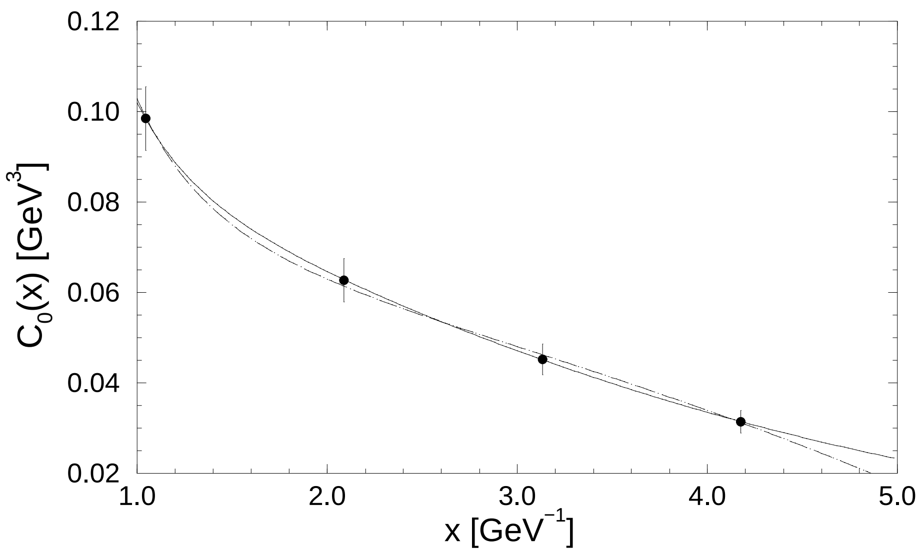

Figure 5: Lattice data from [19] with $a \cdot m_q = 0.01$, quenched calculation. Dashed curve: OPE with condensates of table 1, first row, solid curve: equation (5.6) with $f_P = 0.245\,\mathrm{GeV}^{3/2}$, $E_P = 411\,\mathrm{MeV}$, $w_0^P = 1.0\,\mathrm{GeV}$, $f_S = 0$ and $w_0^S = 0$.

perturbative term was taken into account and no resonance was singled out. Using in addition $m_q = 38\,\mathrm{MeV}$, the fit leads to $f_P = 0.245 \pm 0.002\,\mathrm{GeV}^{3/2}$,

$$E_P \;=\; 411 \pm 3\,\mathrm{MeV}\,, \tag{5.7}$$

and $w_0^P = 1.0 \pm 0.01\,\mathrm{GeV}$. Within the uncertainties, these results are in good agreement to the sum rule determination of the last section.

In other lattice investigations [33,34] of the heavy meson systems the main interest was to determine the leptonic decay constant of the heavy light meson in HQET. In both cases it was found that in order to isolate the ground state contribution one has in some way to suppress the higher state contributions. This was done by two different smearing mechanisms. Though the authors claimed that on the lattice the mass gap E_P is not a physical quantity and therefore divergent in the continuum limit they determined this quantity for their lattice spacing. The difference of the mass gaps $E_S - E_P$ however is a physical quantity and must therefore have a continuum limit. In table 2 we collect the results for these quantities for the different approaches and compare to our findings presented above.

6. Conclusions

In this paper we have investigated the gauge invariant quark correlator. We have set up a relation between the correlator and a corresponding correlator of gauge invariant

Ref.	β	a^{-1} [GeV]	E_P [GeV]	$E_S - E_P$ [GeV]
[34]	5.74	1.53	1.93*	0.69
[34]	6.0	2.03	1.58*	0.49
[33]	6.0	2	1.3*	-
[19]	6.0	1.9	0.31	-
this work L	6.0	1.9	0.41	-
this work SR	-	-	0.45	0.55

Table 2: Pseudoscalar mass E_P and difference between scalar and pseudoscalar mass in different approaches for different values of the strong coupling $\beta = 2N_c/g_s^2$ and lattice spacing a. This work L is an analysis of the data lattice data from [19] according to equation (5.6), this work SR the sum rule analysis of section 4. In the lattice calculations of [33, 34] the pseudoscalar mass is not defined for $a \to 0$.

currents in the heavy quark effective theory. The relevant currents are interpolating fields of pseudoscalar and scalar heavy–light mesons. With the method of QCD sum rules and cooled lattice data it is possible to extract the mass gap between the heavy quark pole mass and the lightest bound states of each current. This mass gap is a physical quantity in the sense that it is scale and scheme independent in perturbation theory to all orders in the strong coupling constant. The resulting values have been found to be $E_P = 450 \pm 150\,\text{MeV}$ for the pseudoscalar state and $E_S = 1.0 \pm 0.2\,\text{GeV}$ for the scalar state. The corresponding correlation lengths are: $a_P = 0.44^{+0.22}_{-0.11}\,\text{fm}$ and $a_s = 0.20^{+0.05}_{-0.03}\,\text{fm}$. For the cooled lattice data of [19, 20] an analysis of a perturbative term plus an exponential gave the correlation length $a_P = 0.63^{+0.21}_{-0.13}\,\text{fm}$; the modified analysis with a single resonance plus continuum of equation (5.6) yields the value $E_P = 411\,\text{MeV}$, corresponding to $a_P = 0.48\,\text{fm}$.

These results can be compared to the spectrum of heavy pseudoscalar mesons. Let us make the very simple assumption that up to corrections of $1/m_Q$ where m_Q is the heavy quark pole mass, the heavy meson mass is equal to the heavy quark mass plus binding energy:

$$E_P - \frac{c}{m_Q} = M_P - m_Q\,. \tag{6.1}$$

Here M_P is the mass of the pseudoscalar meson and c is a constant. Assuming in addition that this relation is valid for both the B and D mesons, we can solve for E_P and c. Using $m_b = 5.0 \pm 0.2\,\text{GeV}$, $m_c = 1.8 \pm 0.2\,\text{GeV}$ and experimental values for M_B and M_D, as central values for E_P and c we obtain:

$$E_P = 400\,\text{MeV} \qquad \text{and} \qquad c = 0.6\,\text{GeV}^2\,. \tag{6.2}$$

The result for E_P is in good agreement with the other determinations presented

above. Nevertheless, we should remark that the latter values are very sensitive to the heavy quark pole mass and with the estimated errors on m_b and m_c, the uncertainty on E_P is of the order of 100%. In addition, from the value of c we see that the assumption of small $1/m_Q$ corrections is not valid in the charm case. Still, we find the very qualitative agreement of our results with the spectrum of pseudoscalar mesons noteworthy.

For lattice simulations of the gauge invariant quark correlator where the higher states were suppressed by a smearing procedure the mass gaps are said to diverge in the continuum limit. Though there is no indication of such a divergence with increasing β (see table 2) the very large values for E_P found in [33] and [34] are definitely not physical. The difference $E_S - E_P$ is however a physical quantity. It has been estimated in [34] and the values listed in table 2 agree with the result from the sum rules better than expected in view of the large errors.

The lattice data for $C_0(x)$ have directly been compared to the operator product expansion in leading order QCD. The results for the continuum values of the quark mass, the quark and the mixed condensates are well compatible with the values determined from other sources. The results from the quenched approximation are nearer to the continuum values than those from full QCD. The known decrease of the modulus of quark condensate with the quark mass is also confirmed by the lattice calculations. The ratio of the mixed to the quark condensate M_0^2 comes out to be the same in the quenched and full QCD. There is some controversy on the dependence of M_0^2 on the quark mass [31, 32]. The lattice data support an increase of M_0^2 with the quark mass which was found in [31]. For distances smaller than approximately 1 fm where the operator product expansion can still be applied a good fit to the non–perturbative part of the correlator is also given by the Gaussian $-\langle\bar{q}q\rangle\exp(-M_0^2x^2/16)$ which has previously been used in the nonlocal sum rule approach [11–13].

In our opinion a direct comparison of lattice QCD simulations with QCD sum rules in the framework of the operator product expansion opens a novel route to augment our knowledge on low–lying hadronic states and non–perturbative QCD. Taking the results of this work as encouraging, we intend to further pursue this approach in the future.

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A. Appendix

The Borel transformation in HQET is defined as

$$\widehat{B}_T \equiv \lim_{-w,n\to\infty} \frac{(-w)^{n+1}}{\Gamma(n+1)} \left(\frac{d}{dw}\right)^n , \qquad T = \frac{-w}{n} > 0 \quad \text{fixed} . \tag{A.1}$$

Using this definition, a central formula for the Borel transformation is found to be

$$\widehat{B}_T \, \frac{1}{(E-w-i\epsilon)^\alpha} = \frac{1}{\Gamma(\alpha)T^{\alpha-1}} \, e^{-E/T} . \tag{A.2}$$

Below, we have collected some analytic formulae for the incomplete Gamma function and the functions $\phi(\alpha, y)$ and $\phi'(\alpha, y)$ defined as

$$\begin{aligned} \phi(\alpha, y) &\equiv \Gamma(\alpha) - \Gamma(\alpha, y) , \\ \phi'(\alpha, y) &\equiv \frac{\partial}{\partial\alpha}\Big(\Gamma(\alpha) - \Gamma(\alpha, y)\Big) , \end{aligned} \tag{A.3}$$

which are helpful for the numerical analysis of the sum rules.

$$\begin{aligned}
\Gamma(2, y) &= e^{-y}(1+y) \\
\Gamma(3, y) &= e^{-y}(2+2y+y^2) \\
\Gamma'(2, y) &= \Gamma(0, y) + e^{-y}[1 + (1+y)\ln y] \\
\Gamma'(3, y) &= 2\Gamma(0, y) + e^{-y}[3 + y + (2+2y+y^2)\ln y] \\
\phi(n, y) &= \Gamma(n) \left(1 - e^{-y} \sum_{k=0}^{n-1} \frac{y^k}{k!}\right) , \qquad n = 1, 2, ... \\
\phi'(\alpha, y) &= \int_0^y dt \;\ln t \; e^{-t}\, t^{\alpha-1} \\
\phi'(1, y) &= -\gamma_E - \Gamma(0, y) - e^{-y} \;\ln y \\
\phi'(2, y) &= 1 - \gamma_E - \Gamma(0, y) - e^{-y}\,(1 + (1+y)\ln y) \\
\phi'(3, y) &= 3 - 2\gamma_E - 2\Gamma(0, y) - e^{-y}\left(3 + y + \left(2 + 2y + y^2\right)\ln y\right) \\
\phi'(4, y) &= 11 - 6\gamma_E - 6\Gamma(0, y) \\
&\quad - e^{-y}\left(11 + 5y + y^2 + \left(6 + 6y + 3y^2 + y^3\right)\ln y\right) .
\end{aligned} \tag{A.4}$$

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